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Electroproduction of two light vector mesons in next-to-leading BFKL: study of systematic effects

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Abstract. The forward electroproduction of two light vector mesons is the first example of a collision process between strongly interacting colorless particles for which the amplitude can be written completely within perturbative QCD in the Regge limit with next-to-leading accuracy. In a previous paper we have given a numerical determination of the amplitude in the case of equal photon virtualities by using a definite representation for the amplitude and a definite optimization method for the perturbative series. Here we estimate the systematic uncertainty of our previous determination, by considering a different representation of the amplitude and different optimization methods of the perturbative series. Moreover, we compare our result for the differential cross section at the minimum $|t|$ with a different approach, based on collinear kernel improvement.

1 Introduction

The BFKL approach [1–4] to strong interactions is expected to describe collision processes with a large center-of-mass energy and with a "hard" enough scale to permit the use of perturbative expansion in the strong coupling α_s. In this approach, both in the leading logarithmic approximation (LLA), which means resummation of leading energy logarithms, all terms $(\alpha_s \ln(s))^n$, and in the next-to-leading approximation (NLA), which means resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$, the (imaginary part of the) amplitude for a large-s hard collision process can be written as the convolution of the Green's function of two interacting reggeized gluons with the impact factors of the colliding particles (see, for example, Fig. 1).

The Green's function is determined through the BFKL equation. The NLA singlet kernel of the BFKL equation has been achieved in the forward case [5, 6], after the long program of calculation of the NLA corrections [7–20] (for a review, see [21]). For the non-forward case the ingredients to the NLA BFKL kernel are known since a few years for the color octet representation in the t-channel [22–26]. This color representation is very important for the check of consistency of the s-channel unitarity with the gluon reggeization, i.e. for the "bootstrap" [27–36]. Recently the non-forward NLA BFKL kernel has been derived also in the singlet color representation, i.e. in the pomeron channel, relevant for physical applications [37, 38].

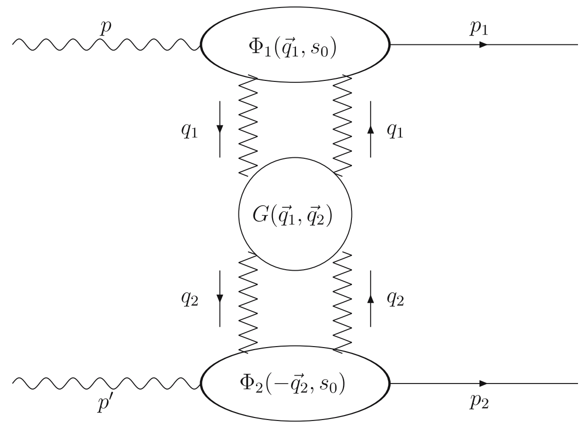

Fig. 1. Schematic representation of the amplitude for the $\gamma^*(p)\gamma^*(p') \to V(p_1)\, V(p_2)$ scattering

On the side of impact factors, however, only limited knowledge is available. Impact factors have been calculated with NLA accuracy for colliding partons [39–41] and for forward jet production [42, 43]. The most important impact factor for the BFKL phenomenology, the $\gamma^* \to \gamma^*$ impact factor, is calling for a rather long calculation, although it seems to be close to completion now [44–51]. The only available colorless impact factor is presently the one for the forward transition from a virtual photon γ^* to a light neutral vector meson $V = \rho^0$, ω, ϕ, obtained in [52, 53]. This impact factor can be used, together with the NLA BFKL Green's function, to build completely

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within perturbative QCD and with NLA accuracy the amplitude of the $\gamma^*\gamma^* \to VV$ reaction. This amplitude provides us with an ideal theoretical laboratory for the investigation of several open questions in the BFKL approach and for the comparison with different approaches.

Such an investigation was started in [54, 55], where it was first of all shown how the $\gamma^* \to V$ impact factors and the BFKL Green's function can be put together to build up the NLA forward amplitude of the $\gamma^*\gamma^* \to VV$ process in the $\overline{\text{MS}}$ scheme and that a convenient series representation for this amplitude can be determined. Then, in the case of equal photon virtualities, i.e. in the so-called "pure" BFKL regime, a numerical study was carried out which led one to conclude that the NLA corrections are large and of opposite sign with respect to the leading order and that they are dominated, at the lower energies, by the NLA correction from impact factors. However, this fact did not prevent us from achieving a smooth behavior of the (imaginary part of the) amplitude with the energy, by optimizing the choice of the energy scale s_0 in the BFKL approach and the renormalization scale μ_R which appear in subleading terms. The optimization method adopted there was an adaptation of the "principle of minimum sensitivity" (PMS) [56, 57] to the case where two energy parameters are present.

The aim of this paper is to estimate the amount of the systematic effects in the determination of [54, 55], by exploring the two main sources: the choice of the representation of the amplitude and the choice of the optimization method. Concerning the first effect, in this paper we compare the series representation of the amplitude with another representation, equivalent to the previous with NLA accuracy, where almost all the NLA corrections coming from the kernel are exponentiated. As for the second effect, we compare here the PMS optimization method with two other well-known methods of optimization of the perturbative series, namely the fast apparent convergence (FAC) method [58–61] and the Brodsky–Lepage–Mackenzie (BLM) method [62].

It would be quite interesting to apply to the amplitude under consideration here the improvement of the NLA BFKL kernel as a consequence of the analysis of collinear singularities of the NLA corrections and by the account of further collinear terms beyond NLA [63–78]. As pointed out in [54, 55], the strategy of collinear improvement has something in common with ours, in the sense that it is also inspired by renormalization-group invariance and it also leads to the addition of terms beyond the NLA. Work is in progress in this direction.

In this paper, however, we present a comparison with an approach by another research group based on the collinear improvement of the kernel. In particular, we compare our determinations for the differential cross section at the minimum $|t|$ of the $\gamma^*\gamma^* \to VV$ process for two values of the common photon virtuality with the results of [79], where the same process has been considered using some version of a collinearly improved kernel.

This paper is organized as follows: in the next section we briefly recall the relevant notation and give the two representations of the amplitude, series and "exponentiated", to be considered in the following; in Sect. 3 we recall the strategies of the three optimization methods considered and perform the numerical comparisons; in Sect. 4 we compare our differential cross section with that of [79]; in Sect. 5 we draw our conclusions.

2 Representations of the NLA amplitude

The process under consideration is the production of two light vector mesons ($V = \rho^0$, ω, ϕ) in the collision of two virtual photons,

$$\gamma^*(p)\gamma^*(p') \to V(p_1)V(p_2)\,. \tag{1}$$

Here, neglecting the meson mass m_V, p_1 and p_2 are taken as Sudakov vectors satisfying $p_1^2 = p_2^2 = 0$ and $2(p_1p_2) = s$; the virtual photon momenta are instead

$$p = \alpha p_1 - \frac{Q_1^2}{\alpha s}p_2\,, \quad p' = \alpha' p_2 - \frac{Q_2^2}{\alpha' s}p_1\,, \tag{2}$$

so that the photon virtualities turn out to be $p^2 = -Q_1^2$ and $(p')^2 = -Q_2^2$. We consider the kinematics when

$$s \gg Q_{1,2}^2 \gg \Lambda_{\text{QCD}}^2\,, \tag{3}$$

and

$$\alpha = 1 + \frac{Q_2^2}{s} + \mathcal{O}(s^{-2})\,, \quad \alpha' = 1 + \frac{Q_1^2}{s} + \mathcal{O}(s^{-2})\,. \tag{4}$$

In this case vector mesons are produced by longitudinally polarized photons in the longitudinally polarized state [52, 53]. Other helicity amplitudes are power suppressed, with a suppression factor $\sim m_V/Q_{1,2}$. We will discuss here the amplitude of the forward scattering, i.e. when the transverse momenta of produced V mesons are zero or when the variable $t = (p_1 - p)^2$ takes its maximal value $t_0 = -Q_1^2Q_2^2/s + \mathcal{O}(s^{-2})$.

The forward amplitude in the BFKL approach may be represented as follows:

$$\begin{aligned}\text{Im}_s(\mathcal{A}) = \frac{s}{(2\pi)^2}\int\frac{\text{d}^2\mathbf{q}_1}{\mathbf{q}_1^2}\Phi_1(\mathbf{q}_1, s_0)\int\frac{\text{d}^2\mathbf{q}_2}{\mathbf{q}_2^2}\Phi_2(-\mathbf{q}_2, s_0)\\ \times\int_{\delta-\text{i}\infty}^{\delta+\text{i}\infty}\frac{\text{d}\omega}{2\pi\text{i}}\left(\frac{s}{s_0}\right)^{\omega}G_\omega(\mathbf{q}_1, \mathbf{q}_2)\,.\end{aligned} \tag{5}$$

This representation for the amplitude is valid with NLA accuracy. Here $\Phi_1(\mathbf{q}_1, s_0)$ and $\Phi_2(-\mathbf{q}_2, s_0)$ are the impact factors describing the transitions $\gamma^*(p) \to V(p_1)$ and $\gamma^*(p') \to V(p_2)$, respectively. The Green's function in (5) obeys the BFKL equation

$$\delta^2(\mathbf{q}_1 - \mathbf{q}_2) = \omega G_\omega(\mathbf{q}_1, \mathbf{q}_2) - \int \text{d}^2\mathbf{q} K(\mathbf{q}_1, \mathbf{q})G_\omega(\mathbf{q}, \mathbf{q}_2)\,, \tag{6}$$

where $K(\mathbf{q}_1, \mathbf{q}_2)$ is the BFKL kernel. The scale s_0 is artificial. It is introduced in the BFKL approach to perform the Mellin transform from the s-space to the complex angular momentum plane and must disappear in the full expression for the amplitude at each fixed order of approximation. Using the result for the meson NLA impact factor, such a cancellation was demonstrated explicitly in [52, 53] for the process in question.

The impact factors are also presented as an expansion in α_s

$$\Phi_{1,2}(\mathbf{q}) = \alpha_s D_{1,2}\left[C^{(0)}_{1,2}(\mathbf{q}^2) + \bar{\alpha}_s C^{(1)}_{1,2}(\mathbf{q}^2)\right],\quad D_{1,2} = -\frac{4\pi e_q f_V}{N_c Q_{1,2}}\sqrt{N_c^2-1}, \tag{7}$$

where f_V is the meson dimensional coupling constant ($f_\rho \approx 200\,\mathrm{MeV}$) and e_q should be replaced by $e/\sqrt{2}$, $e/(3\sqrt{2})$ and $-e/3$ for the case of ρ^0, ω and ϕ meson production, respectively.

In the collinear factorization approach the meson transition impact factor is given as a convolution of the hard scattering amplitude for the production of a collinear quark–antiquark pair with the meson distribution amplitude (DA). The integration variable in this convolution is the fraction z of the meson momentum carried by the quark ($\bar{z} \equiv 1-z$ is the momentum fraction carried by the antiquark):

$$C^{(0)}_{1,2}(\mathbf{q}^2) = \int_0^1 \mathrm{d}z \frac{\mathbf{q}^2}{\mathbf{q}^2 + z\bar{z}Q^2_{1,2}}\phi_\parallel(z). \tag{8}$$

The NLA correction to the hard scattering amplitude, for a photon with virtuality equal to Q^2, is defined as follows

$$C^{(1)}(\mathbf{q}^2) = \frac{1}{4N_c}\int_0^1 \mathrm{d}z \frac{\mathbf{q}^2}{\mathbf{q}^2 + z\bar{z}Q^2}[\tau(z)+\tau(1-z)]\phi_\parallel(z), \tag{9}$$

with $\tau(z)$ given in (75) of [52]. $C^{(1)}_{1,2}(\mathbf{q}^2)$ are given by the previous expression with Q^2 replaced everywhere in the integrand by Q_1^2 and Q_2^2, respectively. We use the distribution amplitude in its asymptotic form $\phi^{\mathrm{as}}_\parallel(z) = 6z(1-z)$. The main reason for this choice is simplicity. The other point is that for the case of equal photon virtualities the typical values of the reggeon momenta are $\mathbf{q}^2 \sim Q^2$. In this case the integrands in (8) and (9) are smooth functions of z and, consequently, the amplitude is not very sensitive to the shape of the meson DA.

In [54, 55] the NLA forward amplitude has been written as a spectral decomposition on the basis of eigenfunctions of the LLA BFKL kernel:

$$\begin{aligned}\frac{\mathrm{Im}_s(\mathcal{A})}{D_1D_2} = \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty}\mathrm{d}\nu\left(\frac{s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(\nu)}\alpha_s^2(\mu_R)\\ \times c_1(\nu)c_2(\nu)\Bigg\{1+\bar{\alpha}_s(\mu_R)\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)}+\frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)\\ +\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{s}{s_0}\right)\left(\bar{\chi}(\nu)+\frac{\beta_0}{8N_c}\chi(\nu)\right.\\ \left.\times\left[-\chi(\nu)+\frac{10}{3}+\mathrm{i}\frac{\mathrm{d}\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{\mathrm{d}\nu}+2\ln\left(\mu_R^2\right)\right]\right)\Bigg\}.\end{aligned} \tag{10}$$

Let us briefly recall the definition of the objects entering this expression:

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi}, \tag{11}$$

with N_c the number of colors;

$$\chi(\nu) = 2\psi(1) - \psi\left(\frac{1}{2}+\mathrm{i}\nu\right) - \psi\left(\frac{1}{2}-\mathrm{i}\nu\right), \tag{12}$$

$$c_1(\nu) = \int \mathrm{d}^2\mathbf{q}\, C_1^{(0)}(\mathbf{q}^2)\frac{(\mathbf{q}^2)^{\mathrm{i}\nu-\frac{3}{2}}}{\pi\sqrt{2}},\quad c_2(\nu) = \int \mathrm{d}^2\mathbf{q}\, C_2^{(0)}(\mathbf{q}^2)\frac{(\mathbf{q}^2)^{-\mathrm{i}\nu-\frac{3}{2}}}{\pi\sqrt{2}}, \tag{13}$$

and similar equations for $c_1^{(1)}(\nu)$ and $c_2^{(1)}(\nu)$ from the NLA corrections to the impact factors, $C_1^{(1)}(\mathbf{q}^2)$ and $C_2^{(1)}(\mathbf{q}^2)$:

$$\begin{aligned}\bar{\chi}(\nu) = -\frac{1}{4}\Bigg[\frac{\pi^2-4}{3}\chi(\nu) - 6\zeta(3) - \chi''(\nu) - \frac{\pi^3}{\cosh(\pi\nu)}\\ +\frac{\pi^2\sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(3+\left(1+\frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right)\\ +4\phi(\nu)\Bigg],\end{aligned} \tag{14}$$

$$\phi(\nu) = 2\int_0^1 \mathrm{d}x\frac{\cos(\nu\ln(x))}{(1+x)\sqrt{x}}\left[\frac{\pi^2}{6} - \mathrm{Li}_2(x)\right],\quad \mathrm{Li}_2(x) = -\int_0^x \mathrm{d}t\frac{\ln(1-t)}{t}. \tag{15}$$

Using (10) we construct the following representation for the amplitude:

$$\begin{aligned}\frac{Q_1Q_2}{D_1D_2}\frac{\mathrm{Im}_s(\mathcal{A}_{\mathrm{series}})}{s} = \frac{1}{(2\pi)^2}\alpha_s(\mu_R)^2\\ \times\left[b_0+\sum_{n=1}^{\infty}\bar{\alpha}_s(\mu_R)^n b_n\left(\ln\left(\frac{s}{s_0}\right)^n\right.\right.\\ \left.\left.+d_n(s_0,\mu_R)\ln\left(\frac{s}{s_0}\right)^{n-1}\right)\right],\end{aligned} \tag{16}$$

where the coefficients

$$\frac{b_n}{Q_1Q_2} = \int_{-\infty}^{+\infty}\mathrm{d}\nu\, c_1(\nu)c_2(\nu)\frac{\chi^n(\nu)}{n!} \tag{17}$$

are determined by the kernel and the impact factors in LLA. The coefficients

$$
\begin{aligned}
d_n &= n \ln\left(\frac{s_0}{Q_1 Q_2}\right) \\
&+ \frac{\beta_0}{4N_c}\left((n+1)\frac{b_{n-1}}{b_n}\ln\left(\frac{\mu_\mathrm{R}^2}{Q_1 Q_2}\right) - \frac{n(n-1)}{2}\right. \\
&+ \frac{Q_1 Q_2}{b_n}\int_{-\infty}^{+\infty} \mathrm{d}\nu (n+1) f(\nu) c_1(\nu) c_2(\nu) \left.\frac{\chi^{n-1}(\nu)}{(n-1)!}\right) \\
&+ \frac{Q_1 Q_2}{b_n}\left(\int_{-\infty}^{+\infty} \mathrm{d}\nu c_1(\nu) c_2(\nu) \frac{\chi^{n-1}(\nu)}{(n-1)!}\right. \\
&\times \left.\left[\frac{\bar{c}_1^{(1)}(\nu)}{c_1(\nu)} + \frac{\bar{c}_2^{(1)}(\nu)}{c_2(\nu)} + (n-1)\frac{\bar{\chi}(\nu)}{\chi(\nu)}\right]\right) \qquad (18)
\end{aligned}
$$

are determined by the NLA corrections to the kernel and to the impact factors. Here $\bar{c}_{1,2}^{(1)}$ are determined according to the definitions (13) from

$$
\begin{aligned}
\bar{C}^{(1)}(\mathbf{q}^2) &= C^{(1)}(\mathbf{q}^2) - \int_0^1 \mathrm{d}z \frac{\mathbf{q}^2}{\mathbf{q}^2 + z\bar{z}Q^2}\phi_\parallel(z) \\
&\times \left[\frac{1}{4}\ln\left(\frac{s_0}{Q^2}\right)\ln\left(\frac{(\alpha + z\bar{z})^4}{\alpha^2 z^2 \bar{z}^2}\right)\right. \\
&\left. + \frac{\beta_0}{4N_c}\left(\ln\left(\frac{\mu_\mathrm{R}^2}{Q^2}\right) + \frac{5}{3} - \ln(\alpha)\right)\right]. \qquad (19)
\end{aligned}
$$

Moreover, we use the notation

$$
\begin{aligned}
f(\nu) &= \frac{5}{3} + \psi(3 + 2\mathrm{i}\nu) + \psi(3 - 2\mathrm{i}\nu) \\
&- \psi\left(\frac{3}{2} + \mathrm{i}\nu\right) - \psi\left(\frac{3}{2} - \mathrm{i}\nu\right). \qquad (20)
\end{aligned}
$$

One should stress that both representations of the amplitude (16) and (10) are equivalent with NLA accuracy, since they differ only by next-to-NLA (NNLA) terms. The series representation (16) is a natural choice, since it includes in some sense the minimal amount of NNLA contributions; moreover, its form is the one closest to the initial goal of the BFKL approach, i.e. to sum selected contributions in the perturbative series.

Actually there exist infinitely many possibilities to write a NLA amplitude. The other possibility considered here is to exponentiate the bulk of the kernel NLA corrections

$$
\begin{aligned}
\frac{\mathrm{Im}_s(\mathcal{A}_\mathrm{exp})}{D_1 D_2} &= \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty} \mathrm{d}\nu \\
&\times \left(\frac{s}{s_0}\right)^{\bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\chi(\nu) + \bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R})\left(\bar{\chi}(\nu) + \frac{\beta_0}{8N_c}\chi(\nu)\left[-\chi(\nu) + \frac{10}{3}\right]\right)} \\
&\times \alpha_\mathrm{s}^2(\mu_\mathrm{R}) c_1(\nu) c_2(\nu)\left[1 + \bar{\alpha}_\mathrm{s}(\mu_\mathrm{R})\left(\frac{c_1^{(1)}(\nu)}{c_1(\nu)} + \frac{c_2^{(1)}(\nu)}{c_2(\nu)}\right)\right. \\
&+ \bar{\alpha}_\mathrm{s}^2(\mu_\mathrm{R})\ln\left(\frac{s}{s_0}\right)\frac{\beta_0}{8N_c}\chi(\nu) \\
&\times \left.\left(\mathrm{i}\frac{\mathrm{d}\ln\left(\frac{c_1(\nu)}{c_2(\nu)}\right)}{\mathrm{d}\nu} + 2\ln\left(\mu_\mathrm{R}^2\right)\right)\right]. \qquad (21)
\end{aligned}
$$

This form of the NLA amplitude was used in [80] (see also [81]), without account of the last two terms in (21), for the analysis of the total $\gamma^*\gamma^*$ cross section. We will refer in the following to this representation simply as "exponentiated" amplitude.

It is easily seen from (16)–(20) that the amplitude is independent in the NLA of the choice of energy and strong coupling scales [54, 55].

For the purposes of our analysis of systematic effects, it could be acceptable also to use an amplitude where the last two terms in the squared brackets of the integrand in the R.H.S. of (21) are exponentiated. Indeed, we performed a numerical analysis also in this case obtaining results which are in fair agreement with our findings below. Nevertheless, we prefer not to exponentiate these terms for the following reason. By exponentiation we mean to transfer the effect from the kernel to the Green's function, which means to account for the corresponding effect "to all orders". The last term in (21) originates from the scale non-invariant part of the NLA kernel, i.e. from the running of the coupling, which leads in Mellin space to the derivative term, see (25) in [54]. It is known [82], that an exact account of the derivative term in the BFKL equation leads to a radical change of both the spectrum and the eigenfunctions of the NLA BFKL kernel. Another point is that the BFKL approach itself (where an amplitude is a convolution of the Green's function and the impact factors) is valid only within NLA. In higher orders one should take into account additional contributions, related, for instance, with the transition of two to four reggeized gluons propagating in the t-channel. In this situation we decided to leave the last two terms in (21) unexponentiated.

Another interesting representation for the BFKL NLA amplitude appears if one treats the energy scale parameters dynamically, allowing s_0 to be a function of the reggeons transverse momenta. Such a change of the energy scale leads to the corresponding modification of the impact factors, and even of the kernel of BFKL equation in the case if the energy scale does not factorize as the product of two functions of $\mathbf{q}_1$ and $\mathbf{q}_2$ [83]. However numerical implementation of such a dynamical energy scale scheme would require knowledge of the NLA BFKL Green's function in the momentum representation, $G(\mathbf{q}_1, \mathbf{q}_2)$, and its subsequent integration with the impact factors. Such a study could be done with the methods developed in [84].

3 Numerical results

In [54, 55] we have presented some numerical results for the amplitude given in (16) for the $Q_1 = Q_2 \equiv Q$ kinematics, i.e. in the "pure" BFKL regime. We truncated the series in the R.H.S. of (16) to $n = 20$, after having verified that this procedure gives a very good approximation of the infinite sum for the Y values $Y \leq 10$ and used the two-loop running coupling corresponding to the value $\alpha_\mathrm{s}(M_Z) = 0.12$.

We obtained there the following results for the b_n and d_n coefficients for $n_f = 5$ and $s_0 = Q^2 = \mu_R^2$:

$$
\begin{aligned}
b_0 &= 17.0664\,, & b_1 &= 34.5920\,, & b_2 &= 40.7609\,,\\
b_3 &= 33.0618\,, & b_4 &= 20.7467\,, & b_5 &= 10.5698\,,\\
b_6 &= 4.54792\,, & b_7 &= 1.69128\,, & b_8 &= 0.554475\,,\\
 & & d_1 &= -3.71087\,, & d_2 &= -11.3057\,,\\
d_3 &= -23.3879\,, & d_4 &= -39.1123\,, & d_5 &= -59.207\,,\\
d_6 &= -83.0365\,, & d_7 &= -111.151\,, & d_8 &= -143.06\,,
\end{aligned} \tag{22}
$$

the main contribution to the d_n coefficients originating from the NLA corrections to the impact factors for $n \leq 3$.

These numbers make visible the effect of the NLA corrections: the d_n coefficients are negative and increasingly large in absolute values as the perturbative order increases. In such a situation it becomes necessary to optimize the perturbative expansion, by proper choice of the renormalization scale μ_R and of the energy scale s_0.

Several ways are known to optimize a perturbative expansion. In [54, 55] we adopted the principle of minimal sensitivity (PMS) [56, 57]. Usually PMS is used to fix the value of the renormalization scale for the strong coupling. We used this principle in a broader sense, requiring the minimal sensitivity of the predictions to the change of both the renormalization and the energy scales, μ_R and s_0. In [54, 55] we considered the amplitude for $Q^2 = 24\,\mathrm{GeV}^2$ and $n_f = 5$ and studied its sensitivity to variation of the parameters μ_R and $Y_0 = \ln(s_0/Q^2)$. We could see that for each value of $Y = \ln(s/Q^2)$ there are quite large regions in μ_R and Y_0 where the amplitude is practically independent on μ_R and Y_0 and we got for the amplitude a smooth behavior in Y, shown in Fig. 2. The optimal values turned out to be $\mu_R \simeq 10Q$ and $Y_0 \simeq 2$, quite far from the kinematical values $\mu_R = Q$ and $Y_0 = 0$. These "unnatural" values are a manifestation of the nature of the BFKL series: NLA corrections are large and then, necessarily, since the exact amplitude should be renormalization and energy scale invariant, the NNLA terms should be large and of the opposite sign with respect to the NLA. These large NNLA corrections are mimicked by the "unnatural" optimal values of μ_R and Y_0.

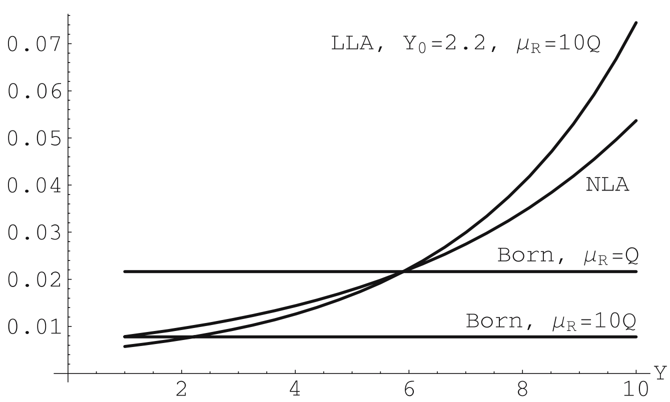

Fig. 2. $\mathrm{Im}_s(\mathcal{A}_{\rm series})Q^2/(sD_1D_2)$ as a function of Y for optimal choice of the energy parameters Y_0 and μ_R (*curve* labeled by "NLA") The *other curves* represent the LLA result for $Y_0 = 2.2$ and $\mu_R = 10Q$ and the Born (two-gluon exchange) limit for $\mu_R = Q$ and $\mu_R = 10Q$. The photon virtuality Q^2 has been fixed to $24\,\mathrm{GeV}^2$ ($n_f = 5$)

As anticipated in the introduction, it is important to have an estimate of the systematic uncertainty which plagues our determination of the energy behavior of the amplitude. The main sources of systematic effects are given by the choice of the representation of the amplitude and by the optimization method adopted. In the following, we compare the determination of the amplitude at $Q^2 = 24\,\mathrm{GeV}^2$ ($n_f = 5$) through the PMS method, given in Fig. 2, with other determinations obtained changing either the representation of the amplitude or the optimization method.

At first, we compare the series and the "exponentiated" determinations using in both cases the PMS method. The procedure we followed to determine the energy behavior of the "exponentiated" amplitude is straightforward: for each fixed value of Y we determined the optimal choice of the parameters μ_R and Y_0 for which the amplitude given in (21) is the one least sensitive to their variation. Also in this case we could see wide regions of stability of the amplitude in the (μ_R, Y_0) plane. The optimal values of μ_R and Y_0 are quite similar to those obtained in the case of the series representation, with only a slight decrease of the optimal μ_R. In Fig. 3 we show the result and compare it to the PMS determination from the series representation. The two curves are in good agreement at the lower energies, the deviation increasing for large values of Y. It should be stressed, however, that the applicability domain of the BFKL approach is determined by the condition $\bar{\alpha}_s(\mu_R)Y \sim 1$ and, for $Q^2 = 24\,\mathrm{GeV}^2$ and for the typical optimal values of μ_R, one gets from this condition $Y \sim 5$. Around this value the discrepancy between the two determinations is within a few percent.

We repeated the same analysis at $Q^2 = 5\,\mathrm{GeV}^2$ ($n_f = 4$) and compared the PMS determination from the 'exponentiated' amplitude with the PMS determination from the series representation, obtained first in our paper [54, 55].

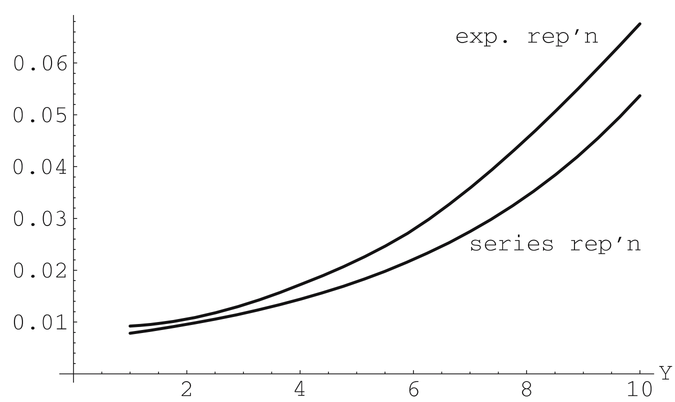

Fig. 3. $\mathrm{Im}_s(\mathcal{A})Q^2/(sD_1D_2)$ as a function of Y at $Q^2 = 24\,\mathrm{GeV}^2$ ($n_f = 5$) from series and "exponentiated" representations, in both cases with the PMS optimization method

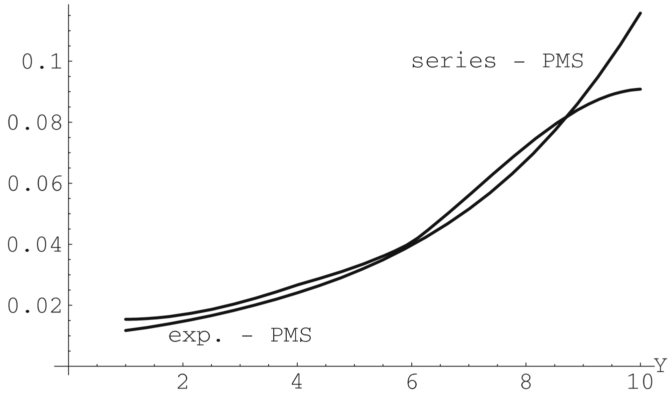

Fig. 4. The same as Fig. 3 at $Q^2 = 5\,\mathrm{GeV}^2$ $(n_f = 4)$

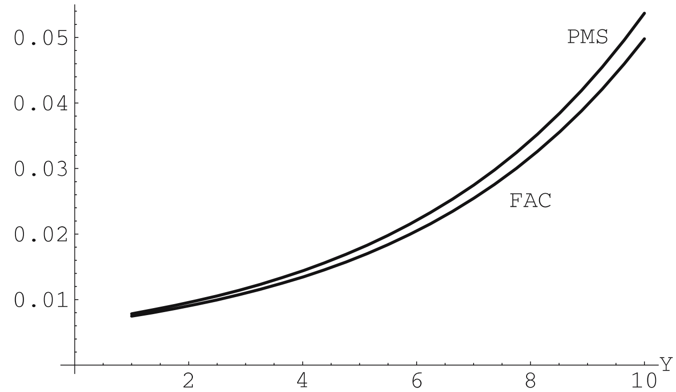

Fig. 5. $\mathrm{Im}_s(\mathcal{A}_{\mathrm{series}})Q^2/(sD_1D_2)$ as a function of Y at $Q^2 = 24\,\mathrm{GeV}^2$ $(n_f = 5)$ from the series representation with PMS and FAC optimization methods

Figure 4 shows that the two determinations are in nice agreement. Despite that, one should stress that in the case $Q^2 = 5\,\mathrm{GeV}^2$ we found for the exponentiated amplitude a much higher value for the optimal energy scale, $Y_0 \simeq 6$. This may be an indication that the convergence of NLA BFKL approach is actually worse for this smaller scale than it is for $Q^2 = 24\,\mathrm{GeV}^2$.

As a second check, we changed the optimization method and applied it both to the series and to the "exponentiated" representation. The method considered is the fast apparent convergence (FAC) method [58–61], whose strategy, when applied to a usual perturbative expansion, is to fix the renormalization scale to the value for which the highest order correction term is exactly zero. In our case, the application of the FAC method requires an adaptation, for two reasons: the first is that we have two energy parameters in the game, μ_{R} and Y_0; the second is that, if only strict NLA corrections are taken, the amplitude does not depend at all on these parameters.

Therefore, in the case of the series representation, (16), we choose to put to zero the sum

$$\frac{1}{(2\pi)^2}\alpha_{\mathrm{s}}(\mu_{\mathrm{R}})^2\sum_{n=1}^{\infty}\bar{\alpha}_{\mathrm{s}}(\mu_{\mathrm{R}})^n b_n d_n(s_0,\mu_{\mathrm{R}})\ln\left(\frac{s}{s_0}\right)^{n-1}$$

and thus found for each fixed Y the values of μ_{R} and Y_0 for which the vanishing occurs. This gives a line of values in the (μ_{R}, Y_0) plane, among which the optimal choice is done applying a minimum sensitivity criterion. The result is shown in Fig. 5. The agreement with the series representation with the PMS method is rather good over a wide energy range.

In the case of the "exponentiated" amplitude, (21), we proceeded in the same way, but requiring the vanishing of the expression given by the R.H.S. of (21) minus the LLA amplitude, i.e.

$$\frac{\mathrm{Im}_s(\mathcal{A}_{\mathrm{exp}})}{D_1D_2} - \frac{s}{(2\pi)^2}\int_{-\infty}^{+\infty}\mathrm{d}\nu\left(\frac{s}{s_0}\right)^{\bar{\alpha}_{\mathrm{s}}(\mu_{\mathrm{R}})\chi(\nu)} \times \alpha_{\mathrm{s}}^2(\mu_{\mathrm{R}})c_1(\nu)c_2(\nu)\,.$$

In Fig. 6 the result is compared with a series representation in the PMS method: there is nice agreement over the whole energy range considered.

Another popular optimization procedure is the Brodsky–Lepage–Mackenzie (BLM) method [62], which amounts to performing a finite renormalization to a physical scheme and then choose the renormalization scale in order to remove the β_0-dependent part. We applied this method only to the series representation, (16), and proceeded as follows: we first performed a finite renormalization to the momentum (MOM) scheme with $\xi = 0$ [80],

$$\begin{aligned}
&\alpha_{\mathrm{s}} \to \alpha_{\mathrm{s}}\left[1 + T_{\mathrm{MOM}}(\xi=0)\frac{\alpha_{\mathrm{s}}}{\pi}\right],\\
&T_{\mathrm{MOM}}(\xi=0) = T_{\mathrm{MOM}}^{\mathrm{conf}} + T_{\mathrm{MOM}}^{\beta},\\
&T_{\mathrm{MOM}}^{\mathrm{conf}} = \frac{N_c}{8}\frac{17}{2}I\,, \quad T_{\mathrm{MOM}}^{\beta} = -\frac{\beta_0}{2}\left[1 + \frac{2}{3}I\right],\\
&I \simeq 2.3439\,;
\end{aligned}$$

then, we chose Y_0 and μ_{R} in order to make the term proportional to β_0 in the resulting amplitude vanish. We observe

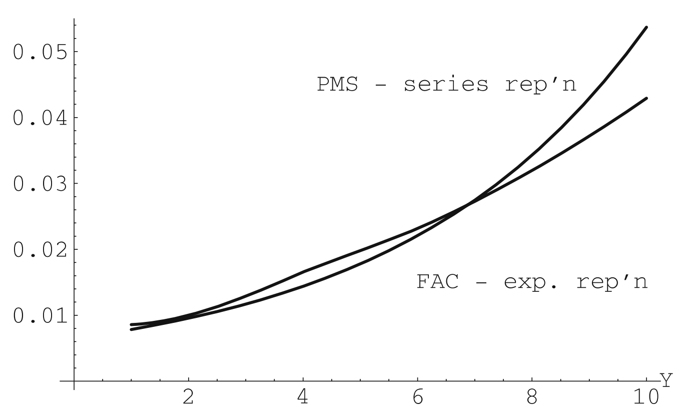

Fig. 6. $\mathrm{Im}_s(\mathcal{A})Q^2/(sD_1D_2)$ as a function of Y at $Q^2 = 24\,\mathrm{GeV}^2$ $(n_f = 5)$ from the series representation with the PMS optimization method and from the "exponentiated" representation with the FAC optimization method

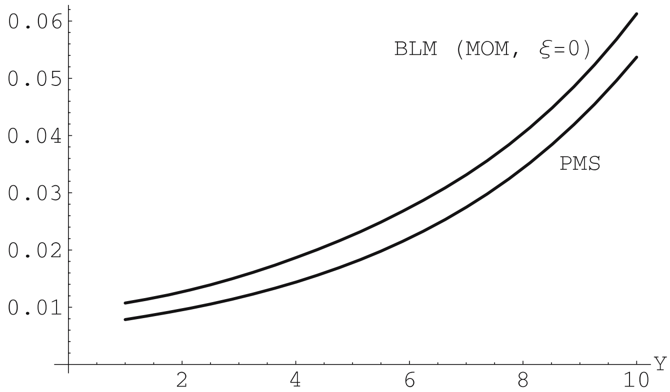

Fig. 7. $\mathrm{Im}_s(\mathcal{A})Q^2/(sD_1D_2)$ as a function of Y at $Q^2 = 24\,\mathrm{GeV}^2$ ($n_f = 5$) from the series representation with PMS and BLM optimization methods

that the β_0-dependence in the series representation of the amplitude is hidden in the d_n coefficients, (18). Among the resulting pairs of values for Y_0 and μ_R, we determined the optimal one according to minimum sensitivity. This method has a drawback in our case, since for each fixed Y, the optimal choice for Y_0 turned out to be always $Y_0 \simeq Y$. However, if one blindly applies the procedure above, one gets a curve which slightly overshoots the one for the series representation with the PMS method; see Fig. 7.

4 The differential cross section at the minimum $|t|$: Comparison with a different approach

The $\gamma^*\gamma^* \to \rho\rho$ process at the lowest order (two-gluon exchange in the t-channel) was studied in [85]. At that level our results coincide; see also [54, 55]. The same process with the inclusion of NLA BFKL effects has been considered in [79]. In that paper, the amplitude has been built with the following ingredients: leading-order impact factors for the $\gamma^* \to \rho$ transition, BLM scale fixing for the running of the coupling in the prefactor of the amplitude (the BLM scale is found using the NLA $\gamma^* \to \rho$ impact factor calculated in [52, 53]) and the renormalization-group-resummed BFKL kernel, with resummation performed on the LLA BFKL kernel at fixed coupling [86, 87]. In [79] the behavior of $\mathrm{d}\sigma/\mathrm{d}t$ at $t = t_0$ was determined as a function of $\sqrt{s}$ for three values of the common photon virtuality, $Q = 2$, 3 and 4 GeV.

In order to make a comparison with the findings of [79], we computed $\mathrm{d}\sigma/\mathrm{d}t$ at $t = t_0$ for $Q = 2$ and $Q = 4$ GeV as functions of $\sqrt{s}$. We used $f_\rho = 216$ MeV, $\alpha_\mathrm{EM} = 1/137$ and the two-loop running strong coupling corresponding to the value $\alpha_\mathrm{s}(M_Z) = 0.12$. The results are shown in the linear-log plots of Figs. 8 and 9, which show a large disagreement. It would be interesting to understand to what extent this disagreement is due to the use in [79] of LLA impact factors instead of the NLA ones or to the way the collinear improvement of the kernel is performed.

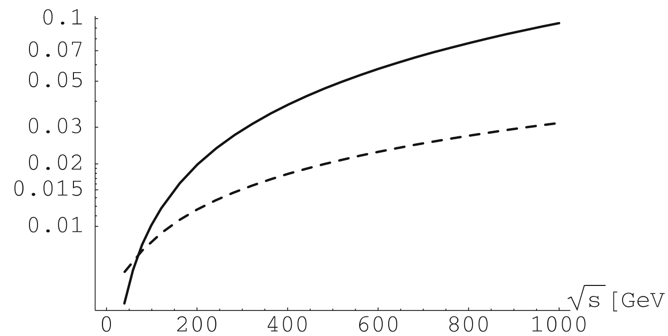

Fig. 8. Linear-log plot of $\mathrm{d}\sigma/\mathrm{d}t|_{t=t_0}$ [pb/GeV^2] as a function of $\sqrt{s}$ at $Q^2 = 16\,\mathrm{GeV}^2$ ($n_f = 4$) from the series representation with the PMS optimization method (*solid line*) compared with the determination from the approach in [79] (*dashed line*)

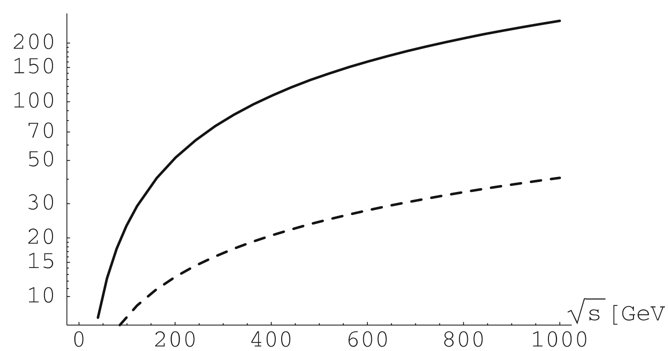

Fig. 9. Linear-log plot of $\mathrm{d}\sigma/\mathrm{d}t|_{t=t_0}$ [pb/GeV^2] as a function of $\sqrt{s}$ at $Q^2 = 4\,\mathrm{GeV}^2$ ($n_f = 3$) from the series representation with the PMS optimization method (*solid line*) compared with the determination from the approach in [79] (*dashed line*)

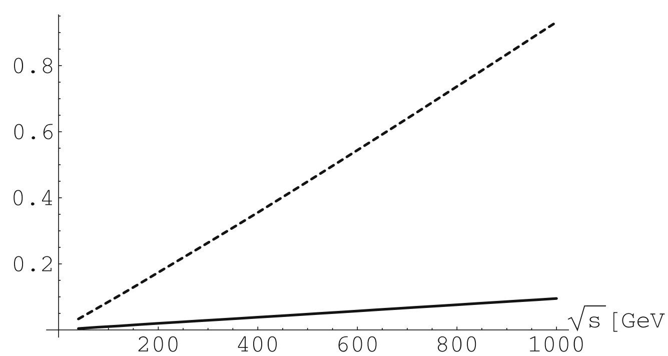

Fig. 10. Linear plot of $\mathrm{d}\sigma/\mathrm{d}t|_{t=t_0}$ [pb/GeV^2] as a function of $\sqrt{s}$ at $Q^2 = 16\,\mathrm{GeV}^2$ ($n_f = 4$) from the series representation with the PMS optimization method using NLO impact factors (*solid line*) and LO impact factors (*dashed line*)

In order to understand to what extent the discrepancy is due to the use of leading-order (LO) impact factors instead of next-to-leading order (NLO) ones, we repeated our determination of $\mathrm{d}\sigma/\mathrm{d}t$ at $t = t_0$ for $Q = 2$ and $Q = 4$ GeV, using LO impact factors and keeping from their NLO contribution only the terms proportional to $\ln[s_0/(Q_1Q_2)]$ and to $\ln[\mu_\mathrm{R}^2/(Q_1Q_2)]$ which are universal and are needed to guarantee the s_0- and μ_R-independence of the amplitude

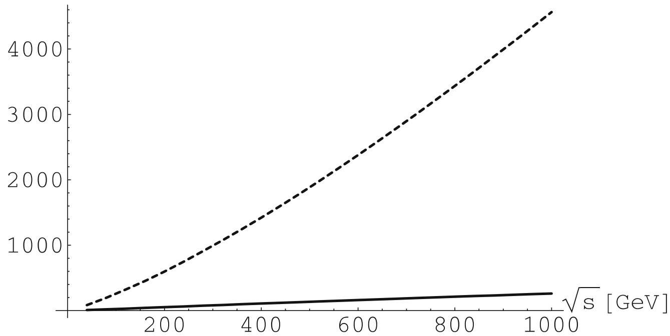

Fig. 11. Linear plot of $\mathrm{d}\sigma/\mathrm{d}t|_{t=t_0}$ [pb/GeV^2] as a function of $\sqrt{s}$ at $Q^2 = 4\,\mathrm{GeV}^2$ ($n_f = 3$) from the series representation with the PMS optimization method (*solid line*) compared with the determination from the approach in [79] (*dashed line*)

with NLA accuracy. The result is that $\mathrm{d}\sigma/\mathrm{d}t$ at $t = t_0$ increases roughly by an order of magnitude with respect to our previous determination (see Figs. 10 and 11) and therefore the disagreement with [79] becomes even worse. This is not surprising: impact factors give a sizable contribution to the NLA part of the amplitude which is negative with respect to the LLA part; if they are kept at LO, the NLA part of the amplitude is less negative, and the total amplitude is therefore increased.

5 Conclusions

We have studied the amplitude for the forward transition from two virtual photons with equal virtuality to two light vector mesons in the Regge limit of QCD with next-to-leading order accuracy. We have found that its behavior with the center-of-mass energy is stable in the applicability region of the BFKL approach under change of representation of the amplitude and under change of the method of optimization of the perturbative series.

We have determined also the differential cross section at the minimum $|t|$ for two values of the common photon virtuality and found strong disagreement with another determination based on the inclusion of next-to-leading order effects only through the kernel and on the use of collinear improvement of the kernel at the leading order.

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